Unit 3 Day 4
Discrete Probability Distributions
(5-3) Binomial Distributions

I. Binomial Formula

A probability experiment that can be reduced to only **2** outcomes is considered a **binomial experiment**.

for ex: Flipping a coin (heads, tails)
Medical Experiment (effective, ineffective)
True/False questions
Games (Win, lose)

And many many others!

I. Binomial Formula

A binomial experiment must satisfy the following four requirements:

1. There must be a fixed # of trials

2. Each trial can only have two outcomes that will be considered a success or failure.

3. The outcomes of each trial must be independent of each other.

4. The probability of a success must remain the same for each trial.

I. Binomial Formula

The outcomes of a binomial experiment and the corresponding probabilities of these outcomes are called a **binomial distribution**.

NOTATION:

p = the numerical prob. of success

q = the numerical prob. of failure = (1-p)

n = number of trials

X = the number of successes in n trials

I. Binomial Formula

In a binomial experiment, the probability of exactly X successes in n trials is:

$$P(X) = \frac{n!}{(n-X)!X!} \bullet p^X \bullet q^{n-X}$$

I. Binomial Formula

Example

1.) A coin is tossed 3 times. Find the probability of getting exactly 2 tails. $p = 0.5$ $q = 0.5$ $n = 3$ $x = 2$

$$P(2) = \left[\frac{(3!)}{(1!\,2!)}\right](0.5)^2(0.5)^1$$

$$P(2) = 3 \cdot (0.5)^2(0.5)^1$$

$$P(2) = 0.375 \longrightarrow \boxed{37.5\%}$$

I. Binomial Formula

Example

2.) A survey found that one out of five Americans say he or she has visited a doctor in any given month. If 10 people are selected at random, find the probability that exactly 3 will have visited a doctor last month.

$p = 1/5 = 0.2$ $\quad n = 10$

$q = 4/5 = 0.8$ $\quad x = 3$

$$P(3) = \left[\frac{(10!)}{(7!\,3!)}\right](0.2)^3(0.8)^7$$

$$P(3) = 120 \cdot (0.2)^3(0.8)^7$$

$$P(3) = 0.2013 \longrightarrow \boxed{20.13\%}$$

I. Binomial Formula

Example

3.) If 23% of all doctors are internists, find the probability that in a group of 15 doctors, 4 are internists.

$p = 0.23 \quad n = 15$

$q = 0.77 \quad x = 4$

$$P(4) = \left[\frac{(15!)}{(11!\,4!)}\right](0.23)^4(0.77)^{11}$$

$$P(4) = 1365 \cdot (0.23)^4 (0.77)^{11}$$

$$P(4) = 0.2155 \longrightarrow \boxed{21.55\%}$$

I. Binomial Formula

Example

4.) A survey found that 30% of teenage consumers receive their spending money from part-time jobs. If 5 teenagers are selected at random, find the probability that **at least** 3 of them will have part-time jobs?

$p = 0.3 \quad q = 0.7 \quad n = 5 \quad x = 3+4+5$

Hint: At least means find the P(3), P(4) and P(5) and then add them together for the total.

$$P(3) = \left[\frac{(5!)}{(2!\,3!)}\right](0.3)^3(0.7)^2 = 0.1323$$

$$P(4) = \left[\frac{(5!)}{(1!\,4!)}\right](0.3)^4(0.7)^1 = 0.02835$$

$$P(5) = \left[\frac{(5!)}{(0!\,5!)}\right](0.3)^5(0.7)^0 = 0.00243$$

$$P(X \geq 3) = P(3) + P(4) + P(5) = 0.1631 \rightarrow \boxed{16.31\%}$$

Calculator Directions:

TI-84

1.) Distr (2nd Vars)

2.) A: binompdf (n, p, x) "exactly"

or B: binomcdf (n, p, x) "at most"

TI-Inspire

1.) spreadsheet icon

2.) menu

5: probability

5: distributions ->

D: binomial pdf "exactly"

or E: binomial cdf (more than one)

Example: About 38% of all students get detention.

a.) What is the probability that in a sample of 15 students, exactly 2 have had detention?

binompdf(15, 0.38, 2) = 0.0303 → 3.03%

b.) What is the probability that in a sample of 15 students, at most 2 have had detention?

* Inspire calculators set bounds at 0 and 2 *

binomcdf(15, 0.38, 2) = 0.0382 → 3.82%

c.) What is the probability that in a sample of 15 students, at least 2 have had detention?

* Inspire calculator set bounds as 2 and 15 *

1 − binomcdf(15, 0.38, 1) = 0.9922 → 99.22%

Assignment:

Unit 3 Day 4 WS (purple; binomial distribution w/ calculator)

Working Model of Game Wednesday 3/11
Project Write up Friday 3/13
Unit 3 Test Friday 3/13